The Tuesday Child Solution

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1 Our problem

I have two children. One is a girl born on a Tuesday. What is the probability I have two girls?

2 Double-girl probability

There are $\binom{2+x-1}{x}$ possible combinations of 2 distinct objects (sexes) into x containers. With a family selection of $x = 2$, we have 3 possible 2-tuples:

$$(boy, boy), (boy, girl), (girl, girl)$$

Thus, the probability of a successful trial with 2 girls is $\frac{1}{3}$.

3 Selection probability

The number of possible combinations of days and families is $7+\binom{2+x-1}{x}$. With a family selection of $x = 2$, we have 28 possible combinations.

Let a child selection be represented as the 2-tuple (d_i, s_i), where d_i is a random selection in the pool

$$(Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday)$$

and s_i is a random selection of the child's sex

$$(boy, girl)$$

An individual trial can thus be represented by the 4-tuple (d_1, s_1, d_2, s_2).

4 Trial

Let $d_1 = Tuesday$ and $s_1 = girl$. There are then $7*\binom{2}{1}$ remaining possibilities for (d_2, s_2) that would constitute a successful trial, which would be satisfied by any selection.

Similarly, for $d_2 = Tuesday$ and $s_2 = girl$, there are $7*\binom{2}{1}$ possibilities for (d_1, s_1) that would constitute a successful trial.

These two selections constitute the entire selection space:

$$2 * [7 * \binom{2}{1}] = [7 + \binom{2+2-1}{1}] = 28.$$

One situation is double-counted: $d_1 = Tuesday, s_1 = girl, d_2 = Tuesday, s_2 = girl$. Accounting for this, our probability of selecting some $s_i = girl$ given some $d_k = Tuesday, s_k = girl$ is given by the following:

$$P = \frac{[7 * \binom{2}{1}] - 1}{7 + \binom{2+2-1}{1} - 1} = \frac{13}{27}.$$

5 Intuitive (maybe?) notes

The fact that we chose Tuesday as the special day is a little arbitrary - the generalized condition "given that the first child was born on x day..." would result in the same reasoning.

When choosing a day and a girl as an initial condition:

- If we choose a girl born on a Tuesday *first*, *any* girl born on *any* day will result in a successful trial.
- A girl born on a Tuesday can be selected in *any order*.
- Two boys will result in a trial that is not considered (as it fails our initial premise that one of the children is a girl).

6 Simulation

The simulation of this experiment can be found here:
`https://gist.github.com/OzuYatamutsu/f46d38a0e739e59ae343`

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Out of 100000 trials:
13842 have girls born on a Tuesday (13.84%)
6606 of those have two girls (47.72%)
Expected probability: 13/27 (0.89% error)
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